On Hölder continuity of solution to elliptic systems & variational integrals

Miroslav Bulíček
Mathematical Institute, Faculty of Mathematics and Physics, Charles University in Prague, Czech Republic
mbul8060@karlin.mff.cuni.cz

We consider minimizers to a variational integral

$$J(u) := \int_\Omega F(u, \nabla u) - f \cdot u \, \mathrm{d}x$$

or weak solution to the corresponding Euler-Lagrange equations in $W_0^{1,p}(\Omega; R^N)$ setting with F having p-growth and coercivity with respect to the second variable. It is well known that generally such solutions for such nonlinear systems of PDE's or minimizers to $J(u)$ may not be smooth and even more they can be discontinuous or unbounded. Up to our best knowledge, the only known result when at least the Hölder continuity of a solution/minimizer can be shown is the so-called Uhlenbeck case $F(\nabla u) \sim |\nabla u|^p$ for large $|\nabla u|$ and related generalizations. Our main goal is to find new structural assumptions on F, that are in principle very far from the Uhlenbeck setting, implying the Hölder continuity for minimizers/solutions. We show that the essential role in our analysis plays the Noether equation, which can be deduced by variations with respect to the internal variable x.

Under the so-called splitting condition we show that any minimizer belongs to $\mathrm{BMO}(\Omega; R^N)$. Moreover, we show that in case that F is u-independent, then it is also Hölder continuous. For F being u dependent, we introduce the so-called one-sided condition and for such F's we show that for $p = 2$ any minimizer is Hölder continuous and for $p \neq 2$, we show that any bounded minimizer is also Hölder continuous. Finally, such results also hold for any (not only bounded) minimizer in case that the principal part of F is strictly convex with respect to the second variable. Finally, we discuss the regularity properties of possible solutions to our system that need not to be minima to $J(u)$.

References

[1] *M. Bulíček, J. Frehse*: $\mathcal{C}^\alpha$-regularity for a class of non-diagonal elliptic systems with p-growth. Calc. Var. Partial Differ. Equ. *43* (2012), 441–462.

[2] *M. Bulíček, J. Frehse, M. Steinhauer*: Everywhere $\mathcal{C}^\alpha$-estimates for a class of nonlinear elliptic systems with critical growth. Adv. Calc. Var. in press 2013.