Random ordinary differential equations and their numerical approximation

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Random ordinary differential equations (RODEs) are pathwise ordinary differential equations that contain a stochastic process in their vector field functions. They have been used for many years in a wide range of applications, but have been very much overshadowed by stochastic ordinary differential equations (SODEs). The stochastic process could be a fractional Brownian motion, but when it is a diffusion process there is a close connection between RODEs and SODEs through the Doss-Sussmann transformation and its generalisations, which relate a RODE and an SODE with the same (transformed) solutions. RODEs play an important role in the theory of random dynamical systems and random attractors.

Classical numerical schemes such as Runge-Kutta schemes can be used for RODEs but do not achieve their usual high order since the vector field does not inherit enough smoothness in time from the driving process. It will be shown how, nevertheless, Taylor expansions of the solutions of RODEs can be obtained when the stochastic process has Hölder continuous sample paths and then used to derive pathwise convergent numerical schemes of arbitrarily high order. Modifications for stiff RODEs will also be discussed and illustrated with an application in epidemiology.

References

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