Global wellposedness of a third order in time nonlinear wave equation arising in high intensity ultrasound (HIU)

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We consider a third order in time equation which arises as a model of wave propagation in viscous thermally relaxing fluids. This third order in time equation displays, even in the linear version, a variety of dynamical behaviors for their solutions that depend on the physical parameters in the equation. These range from non-existence and instability to exponential stability (in time). When neglecting sound diffusivity there is a lack of existence of a semigroup associated with the linear dynamics. More specifically, the corresponding linear dynamics consists of three diffusions: two backward and one forward. When diffusivity of the sound is positive, the linear dynamics is described by a strongly continuous group which is exponentially stable when the ratio of $\frac{sound\ speed \times relaxation\ parameter}{sound\ diffusivity}$ is relatively small, and unstable in the complementary regime. If diffusivity of the sound is positive, the linear dynamics is described by a strongly continuous group. This linear evolution is exponentially stable provided sufficiently large viscous damping is accounted for in the model.

The nonlinear PDE model under consideration displays two important characteristics: (i) it is quasilinear and (ii) it is degenerate in the principal part. The main goal of this talk is to present techniques which enable to cope with this class of problems. One of the important ingredients is the development (see [1]) of stability estimates for the linear *nonautonomuous* evolutions with limited regularity of the coefficients and "group" structure. Discussion of relevant physical parameters leads to an optimization of viscous damping coefficients-hence of decay rates obtained for linear evolutions. Parameters that are responsible for infinite dimensional spectral behavior (essential spectrum) are also identified. Nonlinear estimates for both low and high energies along with the exponential bounds obtained for evolutions allow for application of quasilinear methods in order to obtained global existence of solutions. The theoretical estimates proved in the paper are confirmed by numerical simulations and numerical analysis. Optimization of the spectral parameters lead to a construction of an optimal basin of initial data that guarantee global existence of nonlinear solutions.

This is a joint work with Barbara Kaltenbacher from University of Graz, Austria and Maria Pospieszalska, Division of Inflamation Biology, La Jolla Institute for Allergy and Immunology, La Jolla, Ca.

References

[1] *B. Kaltenbacher, I. Lasiecka, M. K. Pospieszalska*: Well-posedness and exponential decay of the energy in the nonlinear Jordan-Moore-Gibson-Thompson equation arising in high intensity ultrasound. Math. Models Methods Appl. Sci. *22* (2012), 34 p.