

Nondegeneracy of blow-up points for the parabolic Keller-Segel system

Philippe Souplet

Université Paris 13, France

souplet@math.univ-paris13.fr

(Joint work with Noriko Mizoguchi.) We consider the Cauchy and the Neumann problems for the parabolic Keller-Segel type systems

$$(KS) \quad \begin{cases} u_t = \nabla \cdot (\nabla u - u^m \nabla v) \\ \kappa v_t = \Delta v - \lambda v + u, \end{cases}$$

in $n \geq 1$ space dimensions, with $\kappa > 0$ and the chemosensitivity parameter $m > 0$. This class of problems, including in particular the classical Keller-Segel system ($m = 1$), arises in the modeling of chemotactic phenomena. System (KS) is known to admit blow-up solutions under suitable assumptions (cf. Herrero-Velázquez (1997), Winkler (2011), Cieślak-Stinner (2011)). We prove the nondegeneracy of blow-up points, extending to system (KS) a classical result of Giga and Kohn for the nonlinear heat equation $w_t - \Delta w = w^p$. Namely, we show that if T is the blow-up time of (u, v) and if $(T - t)^{1/m} u(x, t)$ is sufficiently small for all (x, t) close to (a, T) , then a is not in the blow-up set of (u, v) .