Recent trends in the half-linear oscillation theory

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We consider the second order differential equation with the one-dimensional p-Laplacian

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x,\ p > 1, \tag{1}$$

with continuous functions r, c and $r(t) > 0$. Due to the Hungarian mathematicians Bihari and Elbert, this equation is now commonly refereed to as the *half-linear equation*, because the solution space of (1) has just one half of the properties which characterize linearity (it is homogeneous, but generally not additive). It is known, see [1], that the classical Sturmian theory for the linear second order Sturm-Liouville differential equation (which is the special case $p = 2$ in (1))

$$(r(t)x')' + c(t)x = 0$$

extends almost verbatim to (1), even if some linear methods cannot be "half-linearized", a typical example is the transformation method. Nevertheless, it turned out that the so-called *modified Riccati technique* well substitutes the missing half-linear transformation theory, see [2]–[5]. We will present some recent results in the half-linear oscillation theory obtained along this line, including open problems and perspectives for the next investigation in this area.

References

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