Asymptotic stability of an underwater pendulum with quadratic damping

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The equation considered in this talk is

$$x'' + h(t)x'|x'| + \omega^2 \sin x = 0,$$

where $h(t)$ is continuous and nonnegative for $t \geq 0$ and ω is a positive real number. This may be regarded as an equation of motion of an underwater pendulum. The damping force is proportional to the square of the velocity. The primary purpose is to establish necessary and sufficient conditions on the time-varying coefficient $h(t)$ for the origin to be asymptotically stable. The phase plane analysis concerning the positive orbits of an equivalent planar system to the above-mentioned pendulum equation is used in order to obtain the main results. The obtained criterion is judged by whether the integral of a particular solution of the first-order quasilinear differential equation

$$u' + h(t)u|u| + 1 = 0$$

is divergent or convergent. Since this quasilinear differential equation cannot be solved in general, it can be said that the presented result is expressed by an implicit condition. Explicit sufficient conditions and explicit necessary conditions are also given for the origin to be asymptotically stable. Finally, our results are extended to be applied to an equation with a nonnegative real-power damping force.

References

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