On a quasilinear Schrödinger equation

Andrzej Szulkin
Department of Mathematics, Stockholm University, Sweden
andrzejs@math.su.se

We consider the quasilinear Schrödinger equation

$$-\Delta u + V(x)u - \Delta(u^2)u = g(x,u), \quad x \in \mathbb{R}^N,$$

where g and V are periodic in $x_1, \ldots, x_N$, $V > 0$, g is odd in u and of subcritical growth in the sense that $|g(x,u)| \le a(1 + |u|^{p-1})$ for some $4 < p < 2 \cdot 2^*$. We show that this equation has infinitely many geometrically distinct solutions in each of the following two cases:

(i) $g(x,u) = o(u)$ as $u \to 0$, $G(x,u)/u^4 \to \infty$ as $|u| \to \infty$, where G is the primitive of g, and $u \mapsto g(x,u)/u^3$ is positive for $u \neq 0$, nonincreasing on $(-\infty, 0)$ and nondecreasing on $(0, \infty)$.

(ii) $g(x,u) = q(x)u^3$, where $q > 0$.

The argument uses the Nehari manifold technique. A special feature here is that the Nehari manifold is not likely to be of class C^1.

This is joint work with Xiangdong Fang.