On travelling waves in nonlinear diffusion models

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We consider the nonlinear evolutionary problem for an unknown function $u = u(x,t)$,

$$\begin{cases} \partial_t u = \operatorname{div}_x(\partial\Phi(\nabla_x u)) + f(u), & x \in \mathbb{R}^N,\ t > 0; \\ u(\cdot,0) = u_0 & \text{in } \mathbb{R}^N. \end{cases} \tag{1}$$

Here, $\Phi\colon \mathbb{R}^N \to \mathbb{R}$ is a p-Laplacian-type nonlinearity, $f\colon \mathbb{R} \to \mathbb{R}$ is of Fisher-Kolmogorov-type, and $u_0\colon \mathbb{R}^N \to \mathbb{R}$ are given data between the two extremal zeros ∓ 1 of f. Under some "natural" hypotheses on the nonlinearities, we first derive the existence and uniqueness of a *travelling wave* for $N = 1$ (valued in $(-1,1)$ or $[-1,1]$), then discuss applications to (i) the existence and stability of spherically symmetric waves and (ii) finite-time extinction of one of the two competing species in the Fisher-Kolmogorov model (by *interaction* between the nonlinear diffusion and reaction).