Periodic oscillations in a singular equation modelling valveless pumping

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The phenomenon of "valveless pumping" (the conveyance of liquid fluids in mechanical systems that have no valves to ensure the preferential direction of the flow) can be found in several physical systems, for instance, the blood circulation in human embryos and invertebrates.

In [1] it was presented an explanation of the pumping effect for flow configurations of 1–3 rigid tanks connected by rigid pipes. These configurations were modeled by systems of ordinary differential equations with quadratic terms for the pipe-tank junctions.

The configuration consisting of 1 pipe and 1 tank is apparently the simplest one and it leads to the following periodic boundary value problem

$$\begin{cases} u'' + au' = \frac{1}{u}(e(t) - b(u')^2) - c, \\ u(0) = u(T),\ u'(0) = u'(T), \end{cases} \tag{1}$$

where, according to the physical meaning of the involved parameters, we may assume $a > 0$, $b > 1$, $c > 0$ and $e(t)$ is continuous and T-periodic. However the singularity that appears on the right-hand side of the differential equation makes it difficult to deal with and, in fact, there is a lack of general existence results for problem (1). Our aim is to fill this gap providing an existence and stability result for quite general periodic forcing terms.

This is a work in progress with G. Propst, M. Tvrdý and M. Zima.

References

[1] *G. Propst*: Pumping effects in models of periodically forced flow configurations. Physica D *217* (2006), 193–201.