Positive solutions of two-point boundary value problems for higher order nonlinear differential equations

Ivan Kiguradze
A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Georgia
kig@rmi.ge

The differential equation

$$u^{(n)} = f(t, u) \tag{1}$$

with the boundary conditions

$$u^{(i-1)}(a) = 0 \quad (i = 1, \dots, m), \quad u^{(k-1)}(b) = 0 \quad (k = 1, \dots, n-m) \tag{2}$$

or

$$u^{(i-1)}(a) = 0 \quad (i = 1, \dots, m), \quad u^{(m+k-1)}(b) = 0 \quad (k = 1, \dots, n-m) \tag{3}$$

is considered. Here $n \geq 2$, $m \in \{1, \dots, n-1\}$, and $f :]a, b[\times]0, +\infty[\to R$ is a continuous function, at that the possibility of

$$\limsup_{x \to +\infty} |f(t, x)| = +\infty$$

and

$$\int_a^b (t-a)^{n-1}(b-t)^{n-1}|f(t, x)|\mathrm{d}t = +\infty$$

is admitted.

Unimprovable in a certain sense conditions are found under which either of problems (1), (2) and (1), (3) has a positive on $]a, b[$ solution.

Acknowledgements. Supported by the Shota Rustaveli National Science Foundation (Project # FR/317/5-101/12).