Boundary value problems with state-dependent impulses

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This is a joint work with Jan Tomeček (Palacký University, Olomouc). Impulsive differential equations have attracted lots of interest due to their important applications in many areas such as aircraft control, drug administration, and threshold theory in biology. A particular case of impulsive problems are *problems with impulses at fixed moments*. This occurs when the moments, at which impulses act in state variable, are known. Very different situation arises, when the impulses appear in evolutionary trajectories fulfilling a predetermined relation between state and time variables. This case, which is represented by state-dependent impulses, is discussed here. In particular, we investigate the solvability of *boundary value problems with state-dependent impulses*. As the methods used for problems with finitely many impulses acting at fixed points do not apply to problems with state-dependent impulses, only few paper dealing with state-dependent case may be found in the literature. Most of them consider periodic problems which can be transformed to fixed point problems of corresponding Poincaré maps. So, in the case of a periodic boundary conditions, difficulties with the construction of a proper function space and a proper operator representation have been cleared, see e.g. [1].

The main cause of difficulties in the investigation of problems with state-dependent impulses lies in the following fact: the operator, corresponding to the problem with state-dependent impulses which is constructed in a standard way (used for problems with fixed-time impulses), is not continuous. Therefore, in [2] and [3] we provide a new approach which makes possible to find sufficient conditions for solvability of the problem

$$z''(t) = f(t, z(t), z'(t)) \quad \text{for a.e. } t \in [a, b],$$

$$z(\tau_i+) - z(\tau_i) = J_i(\tau_i, z(\tau_i)), \quad z'(\tau_i+) - z'(\tau_i-) = M_i(\tau_i, z(\tau_i)), \quad \ell(z, z') = c,$$

where the points $\tau_1, \dots, \tau_p$ depend on z through the equations

$$\tau_i = \gamma_i(z(\tau_i)), \quad i = 1, \dots, p, \quad p \in N.$$

Here f fulfils the Carathéodory conditions, the impulse functions J_i, M_i, and the barriers γ_i, $i = 1, \dots, p$, are continuous, $c \in R^2$, and ℓ is a linear and bounded operator.

References

[1] *I. Bajo, E. Liz*: Periodic boundary value problem for first order differential equations with impulses at variable times. J. Math. Anal. Appl. *204* (1996), 65–73.

[2] *I. Rachůnková, J. Tomeček*: New approach to BVPs with state-dependent impulses. Boundary Value Problems 2013, *2013*:22.

[3] *I. Rachůnková, J. Tomeček*: Second order BVPs with state-dependent impulses via lower and upper functions. To appear in Cent. Eur. J. Math.