Extending Korn's first inequality to incompatible tensor fields

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We prove a Korn-type inequality in $H_0(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$ for tensor fields P mapping Ω to $\mathbb{R}^{3\times 3}$. More precisely, let $\Omega\subset\mathbb{R}^3$ be a bounded domain with connected Lipschitz boundary $\partial\Omega$. Then, there exists a constant $c>0$ such that

$$c\|P\|_{L^2(\Omega,\mathbb{R}^{3\times 3})}\leq\|\operatorname{sym}P\|_{L^2(\Omega,\mathbb{R}^{3\times 3})}+\|\operatorname{Curl}P\|_{L^2(\Omega,\mathbb{R}^{3\times 3})} \tag{1}$$

holds for all tensor fields $P\in H_0(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$, i.e., all $P\in H(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$ with vanishing tangential trace on $\partial\Omega$. Here, rotation and tangential trace are defined row-wise. For compatible P, i.e., $P=\nabla v$ and thus $\operatorname{Curl}P=0$, where $v\in H^1(\Omega,\mathbb{R}^3)$ are vector fields having components v_n, for which ∇v_n are normal at $\partial\Omega$, the presented estimate (1) reduces to a non-standard variant of Korn's first inequality, i.e.,

$$c\|\nabla v\|_{L^2(\Omega,\mathbb{R}^{3\times 3})}\leq\|\operatorname{sym}\nabla v\|_{L^2(\Omega,\mathbb{R}^{3\times 3})}.$$

On the other hand, for skew-symmetric P, i.e., $\operatorname{sym}P=0$, inequality (1) reduces to a non-standard version of Poincaré's estimate. Therefore, since (1) admits the classical boundary conditions our result is a common generalization of the two classical estimates, namely Poincaré's resp. Korn's first inequality. I indicate applications of the new inequality in gradient plasticity with plastic spin.

This is a joint work with Dirk Pauly and Karl-Josef Witsch (Essen).

References

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[2] *P. Neff, D. Pauly, K. J. Witsch*: A canonical extension of Korn's first inequality to $H(\mathrm{Curl})$ motivated by gradient plasticity with plastic spin. C. R., Math., Acad. Sci. Paris *349* (2011), 1251–1254.