First-order system finite elements for nonlinear models in solid mechanics

Gerhard Starke
Faculty of Mathematics, University of Duisburg-Essen, Germany
gerhard.starke@uni-due.de

We review different first-order system approaches to nonlinear models in solid mechanics. In particular, we consider elasto-plastic and hyperelastic material models and discuss the challenges that the nonlinearity poses on the finite element treatment. Methods which simultaneously approximate stresses and displacements are studied in detail with a focus on momentum balance accuracy. First-order system least squares approaches are particularly attractive since they allow the coupling of appropriate H(div)-conforming finite elements for the stress approximation with standard conforming elements for the displacement approximation without restrictions by compatibility conditions on the discrete spaces. In this context, an investigation is done under which circumstances the optimal convergence behavior known from the linear elasticity case carries over to nonlinear models. This is possible, for instance, in the context of elasto-plasticity using analytical properties of the underlying first-order system. On this basis, a posteriori estimation of the error associated with the finite element approximation can be studied and the resulting adaptive computations are shown to behave in an optimal way. In the context of hyperelastic material models, only little is known about the convergence properties of finite element methods. Our aim is to provide insight into the analytical structure of the model by our numerical computations based on a first-order system formulation. Another issue which can be treated within the first-order system context is concerned with the estimation of the modelling error measuring a simpler model like, e.g. linear elasticity with respect to more complicated nonlinear reference models.

References

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