

## Existence results for equations with reflection

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This talk is devoted to the study of the following first order equation with reflection in the argument and coupled with periodic boundary value conditions:

$$x'(t) = f(t, x(t), x(-t)), \quad \text{for a.e. } t \in [-T, T], \quad x(-T) = x(T).$$

To this end, we consider the linear equation

$$x'(t) + a(t)x(t) + b(t)x(-t) = h(t), \quad \text{for a.e. } t \in [-T, T], \quad x(-T) = x(T).$$

We obtain some estimations on the norm of  $a$  and  $b$  (optimal in some cases) to ensure that the linear problem has a unique solution and it has constant sign on  $[-T, T]$ . In this way, we automatically establish maximum and anti-maximum principles for the linear operator. In some particular situations we are able to obtain the exact expression of the Green's function.

The existence results for the nonlinear problem follow from iterative techniques and fixed point theorems in cones. In some of the given results, the Green's function is allowed to change sign on its square of definition.

### References

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