

# On homoclinic solutions of semilinear $p$ -Laplacian difference equations

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We consider the semilinear eigenvalue  $p$ -Laplacian difference equation

$$\Delta_p^2 u(k-1) - V(k)u(k)|u(k)|^{q-2} + \lambda f(k, u(k)) = 0,$$

and looking for its homoclinic solutions, i.e., solutions of the equation such that  $u(k) \rightarrow 0$  as  $|k| \rightarrow \infty$ . Here  $\lambda > 0$ ,  $\{u(k) : k \in \mathbf{Z}\}$  is a sequence of real numbers,  $\Delta u(k) = u(k+1) - u(k)$  and  $\Delta_p^2 u(k-1) = \Delta u(k)|\Delta u(k)|^{p-2} - \Delta u(k-1)|\Delta u(k-1)|^{p-2}$  is referred as the  $p$ -Laplacian difference operator. Under suitable periodic and growth conditions on the functions  $V$  and  $f$  we derive the existence of homoclinic solutions. We prove the main result, published in [1], using variational approach and Brezis-Nirenberg mountain-pass theorem. Three examples of equations are given, arising in mathematical physics and biology, as follows:

(A) Second-order discrete  $p$ -Laplacian equations of the form  $\Delta_p^2 u(k-1) - V(k)u(k)|u(k)|^{q-2} + \lambda b(k)u(k)|u(k)|^{r-2} = 0$  with  $r > p \geq q > 1$ .

(B) Higher even-order difference equations. A model equation is the fourth-order extended Fisher-Kolmogorov equation  $\Delta^4 u(k-2) - a\Delta^2 u(k-1) + V(k)u(k)|u(k)|^{q-2} - \lambda b(k)u(k)|u(k)|^{r-2} = 0$  with  $r > q > 1$ .

(C) Second-order difference equations with cubic and quintic nonlinearities of the form

$$\Delta^2 u(k-1) - V(k)u(k) + \lambda(b(k)u^3(k) + c(k)u^5(k)) = 0.$$

Further, we study also a problem for equation with variable periodic exponents

$$\Delta_{p(k-1)}^2 u(k-1) - V(k)|u(k)|^{q(k)-2}u(k) + f(k, u(k)) = 0,$$

and  $u(k) \rightarrow 0, |k| \rightarrow \infty$ . The results are published in [2].

Next, we consider the case when the potential  $V$  is a coercive function and the existence of multiple homoclinic solutions.

## References

- [1] A. Cabada, C. Li, S. Tersian: On Homoclinic Solutions of a Semilinear  $p$ -Laplacian Difference Equation with Periodic Coefficients, Hindawi Publishing Corporation, Advances in Difference Equations, Volume 2010, Article ID 195376, 17 pages doi:10.1155/2010/195376.
- [2] M. Mihăilescu, V. Rădulescu, S. Tersian: Homoclinic solutions of difference equations with variable exponents. Topological Methods in Nonlinear Analysis, Journal of the Juliusz Schauder University Centre, Volume 38, 2011, 277–289.