Discontinuous gradient constraints and the infinity Laplacian

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Motivated by tug-of-war games and asymptotic analysis of certain variational problems, we consider the following gradient constraint problem: given a bounded domain Ω, a continuous function $f\colon \partial\Omega \to R$ and a non-empty subset $D \subset \Omega$, find a solution to

$$\begin{array}{ll} \min\{\Delta_\infty u, |Du| - \chi_D\} = 0 & \text{in } \Omega \\ u = f & \text{on } \partial\Omega, \end{array}$$

where Δ_∞ is the infinity Laplace operator. We prove that this problem always has a solution that is unique if $\overline{D} = \overline{D^0}$. If this regularity condition on D fails, then solutions obtained from game theory and L^p-approximation need not coincide.

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