Concentration profiles for Moser-Trudinger functional are shaped like toy pyramids

Cyril Tintarev
Uppsala University, Department of Mathematics, Uppsala, Sweden
tintarev@math.uu.se

Let $u_k \rightharpoonup u$ be a Palais-Smale sequence for the semilinear elliptic functional $J(u) = \int_\Omega |\nabla u|^2 - \int_\Omega \Psi(u)$ where $\Omega \subset \mathbb{R}^2$ is a bounded domain and $\Psi(u) \sim e^{4\pi u^2}$ is a nonlinearity of critical growth. Convergence of the critical sequence under general conditions, at levels $c < 1$ of J, was proved by Adimurthi (Annales SNS Pisa, 1990), divergent critical sequences at $c \in \mathbb{N}$ were constructed by Adimurthi and Prashanth, and convergence for sequences of solutions was verified by Druet for all positive $c \notin \mathbb{N}$, which lead to the Adimurthi-Struwe conjecture, that this is also the case for general Palais-Smale sequences. We disprove this conjecture and identify the (multiple) counterpart of the Talenti solution in dimension 2.

Defect of convergence of u_k in the Sobolev norm can be represented as a sum of concentrating terms of the form $\sqrt{t_k} w(|z-\zeta|^{1/t_k})$, $t_k \to \infty$, $\zeta \in \Omega$, where w is a solution of the asymptotic equation under the logarithmic rescaling. We show that such solutions are radial, and that, unlike in the case $N \geq 3$, for any $c > 1$, there are uncountably many distinct (not scaling-equivalent) w defining a Palais-Smale sequence $u_k = \sqrt{t_k} w(|z-\zeta|^{1/t_k}) + o(1)$ such that $J(u_k) \to c$. Furthermore, every such w has Lipschitz continuous derivative, is constant in the neighborhood of the origin, does not exceed $\sqrt{\frac{\log \frac{1}{r}}{2\pi}}$, and on every connected component of $\left\{u(r) < \sqrt{\frac{\log \frac{1}{r}}{2\pi}}\right\}$ is a linear combination of 1 and $\log r$. The method of the proof is abstract concentration theorem of the speaker, whose other applications, with non-standard gauges, include Strichartz imbeddings by Terence Tao and Sobolev imbeddings on periodic manifolds and Lie groups. This is a joint work with David Costa.