Nodal count of graph eigenfunctions as an index of instability

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Zeros of vibrational modes have been fascinating physicists for several centuries. Mathematical study of zeros of eigenfunctions goes back at least to Sturm, who showed that, in dimension $d = 1$, the n-th eigenfunction has $n-1$ zeros. Courant showed that in higher dimensions only half of this is true, namely zero curves of the n-th eigenfunction of the Laplace operator on a compact domain partition the domain into at most n parts (which are called "nodal domains").

It recently transpired (first on graphs with a subsequent generalization to manifolds) that the difference between this upper bound and the actual value can be interpreted as an index of instability of a certain energy functional with respect to suitably chosen perturbations. We will discuss two examples of this phenomenon: (1) stability of the nodal partitions with respect to a perturbation of the partition boundaries and (2) stability of an eigenvalue with respect to a perturbation by magnetic field. In both cases, the "nodal defect" of the eigenfunction coincides with the Morse index of the energy functional at the corresponding critical point.

References

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