On discreteness of spectrum of a functional-differential operator on axis

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Discreteness of spectrum of the functional differential operator

$$\mathcal{L}u = -u'' + p(x)u(x) + \int_R (u(x) - u(s))\,\mathrm{d}_s r(x,s),\ \ x \in R, \tag{1}$$

$R = (-\infty, \infty)$, is studied. It is assumed that the corresponding bilinear form

$$[u,v] = \int_R (u'v' + puv)\,\mathrm{d}x + \frac{1}{2}\int_{R\times R} (u(s) - u(x))(v(s) - v(x))\,\mathrm{d}\xi \tag{2}$$

is symmetric: $\xi(x,s) = \xi(s,x)$. Consider a particular case,

$$\mathcal{L}u = -u'' + p(x)u(x) + q(x)(u(x) - u(x-\delta)) + q(x+\delta)(u(x) - u(x+\delta)), \tag{3}$$

$x \in (-\infty, +\infty)$, where $\delta > 0$ is a constant. Let $\lambda_0(\Delta)$ be the minimal eigenvalue of the *truncated* problem ($u(x) = 0$ if $x \notin \Delta$)

$$\begin{cases} \mathcal{L}_\Delta u = \lambda u, \\ u'(a) = u'(b) = 0. \end{cases} \tag{4}$$

The following assertion is similar to the result of Ismagilov [1] for ordinary differential equation.

Theorem 1 *For discreteness of the spectrum of* (3) *it is necessary and under supplement condition* $\operatorname{ess\,sup}_{x\in R} \frac{q(x)}{p(x)} = M < \infty$ *is sufficient that for any sequence of segments* Δ_n *of fixed length that tends to infinity*

$$\lim \lambda_0(\Delta_n) = \infty. \tag{5}$$

References

[1] *R. S. Ismagilov*: About conditions of semi-boundedness and discreteness of the spectrum for one dimensional differential operators. DAN SSSR *140* (1961), 33–36 (Russian).