Second order quasilinear functional equations

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We shall consider problems of the form

$$u''(t) + [N(t, u'(t); u', u)](t) + Q[u(t)] + [M(t, u'(t); u', u)](t) = f(t), \quad t \in (0, T)$$
$$u(0) = u_0 \in V, \quad u'(0) = u_1 \in H$$

where $1 < p < \infty$,

$$N : L^p(0, T; V) \times [L^p(0, T; V_1)]^2 \to L^q(0, T; V^\star),$$
$$M : L^p(0, T; V) \times [L^p(0, T; V)]^2 \to L^q(0, T; V_1^\star),$$

are demicontinuous bounded nonlinear operators, $Q : V \to V^\star$ is a linear continuous operator, V, V_1 are reflexive Banach spaces such that $V \subset V_1$ and the imbedding is compact, finally, $V \subset H \subset V^\star$ is an evolution triple.

Conditions will be formulated which imply the existence of solutions for $t \in (0, T)$ and for $t \in (0, \infty)$. Further, the boundedness of u', u for $t \in (0, \infty)$ and the stabilization of u as $t \to \infty$ will be shown. Several applications will be considered.

References

[1] *L. Simon*: Application of monotone type operators to parabolic and functional parabolic PDE's. Chapter 6 in Handbook on Evolutionary Differential Equations, Elsevier, 2008.

[2] *L. Simon*: Nonlinear second order evolution equations with state-dependent delays. EJQTDE, Proc. 9th Coll. QTDE, No. 14 (2012), 1–12.