On convergent numerical schemes for two-phase flow of incompressible fluids with different mass densities

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In this talk, we will be concerned with convergence results of numerical schemes for diffuse interface models for two-phase flow of immiscible, incompressible viscous fluids with different mass densities. In contrast to the case of identical mass densities, for general mass densities only recently diffuse interface models have been suggested which are consistent with thermodynamics and which allow for a solenoidal velocity field (see Abels, Garcke, Grün M3AS 2012). These models consist of a new momentum equation for the velocity field coupled to a Cahn-Hilliard equation for the evolution of the order parameter.

A subtle discretization of the convective coupling between the flux of the phase-field and the momentum equation allows to formulate a numerical scheme which satisfies a discrete counterpart of the energy estimate. By higher regularity results for discrete solutions of convective Cahn-Hilliard equations, we prove its convergence in two and in three space dimensions.

Finally, we shall present numerical simulations to underline the full practicality of our approach and to identify physical settings for which the new coupling term suggested in (Abels, Garcke, Grün, M3AS 2012) seems to be indispensable for numerical stability.