

On the existence and stability of solitary-wave solutions to a class of evolution equations of Whitham type

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We consider a class of pseudodifferential evolution equations of the form

$$u_t + (n(u) + Lu)_x = 0,$$

in which L is a linear smoothing operator and n is at least quadratic near the origin; this class includes in particular the Whitham equation, the linear terms of which match the dispersion relation for gravity water waves on finite depth. A family of solitary-wave solutions is found using a constrained minimisation principle and concentration-compactness methods for noncoercive functionals. The solitary waves are approximated by (scalings of) the corresponding solutions to partial differential equations arising as weakly nonlinear approximations; in the case of the Whitham equation the approximation is the Korteweg-deVries equation. We also demonstrate that the family of solitary-wave solutions is conditionally energetically stable.

References

- [1] *M. Ehrnström, M. D. Groves, E. Wahlén*: On the existence and stability of solitary-wave solutions to a class of evolution equations of Whitham type. *Nonlinearity* 25 (2012), 1–34.