Hamiltonian PDEs and dispersive shock waves

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We review the theory of dispersive shock waves and describe it in detail for the small dispersion limit of the Korteweg de Vries equation [1], [2], [3]. Then we discuss some results concerning two component Hamiltonian systems and formulate some conjecture regarding the asymptotic behaviour of the solutions [2]. In particular we argue that the asymptotic behaviour of the solution near critical points is described in terms of ODEs of the Painlevé family. Such local behaviour is universal in the sense that it does not depend on the equation or the initial data, [4], [5], [6].

References

[1] *T. Grava, C. Klein*: A numerical study of the small dispersion limit of the Korteweg-de Vries equation and asymptotic solutions. Physica D *241* (2012), 2246–2264.

[2] *B. Dubrovin, T. Grava, C. Klein*: Numerical Study of breakup in generalized Korteweg-de Vries and Kawahara equations. SIAM J. Appl. Math. *71* (2011), 983–1008.

[3] *T. Claeys, T. Grava*: Universality of the break-up profile for the KdV equation in the small dispersion limit using the Riemann-Hilbert approach. Commun. Math. Phys. *286* (2009), 979–1009.

[4] *B. Dubrovin, T. Grava, C. Klein*: On universality of critical behavior in the focusing nonlinear Schrödinger equation, elliptic umbilic catastrophe and the Tritronquée solution to the Painlevé-I equation. J. Nonlinear Sci. *19* (2009), 57–94.

[5] *B. Dubrovin*: On Hamiltonian perturbations of hyperbolic systems of conservation laws. II: Universality of critical behaviour. Commun. Math. Phys. *267* (2006), 117–139.

[6] *B. Dubrovin*: On universality of critical behaviour in Hamiltonian PDEs. Providence, RI: American Mathematical Society Transl. Ser. 2, 224. Advances in the Mathematical Sciences 61, (2008), pp. 59–109.