Some recent results on regularity of weak solutions to the Navier-Stokes equations

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We give a brief survey of some recent results on the regularity of weak solutions to the Navier-Stokes equations. Then we focus in greater detail on the two regularity criteria which are applicable to the so called suitable weak solutions:

1) the local criterium for regularity at a space-time point $(\mathbf{x}_0, t_0)$, which imposes the Serrin-type integrability condition on the velocity only in a backward neighbourhood of $(\mathbf{x}_0, t_0)$ intersected with the exterior of a certain space-time paraboloid with vertex at point $(\mathbf{x}_0, t_0)$,

2) the criterium which uses assumptions on a spectral projection of vorticity, respectively only on one component of this spectral projection. The projection is defined by means of the spectral resolution of identity associated with the self-adjoint operator **curl** in $\mathbf{R}^3$. We discuss connection with the helicity and the importance of the "positive Beltrami flows" as well as "negative Beltrami flows" with "large modes".

References

[1] *J. Neustupa*: A refinement of the local Serrin-type regularity criterion for a suitable weak solution to the Navier-Stokes equations. Submitted.

[2] *J. Neustupa, P. Penel*: Regularity of a weak solution to the Navier-Stokes equations via one component of a spectral projection of vorticity. Submitted.