Mass transport problems for the Euclidean distance obtained as limits of p-Laplacian type problems with obstacles

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In this talk we analyze a mass transportation problem that consists in moving optimally (paying a transport cost given by the Euclidean distance) an amount of a commodity larger or equal than a fixed one to fulfil a demand also larger or equal than a fixed one, with the obligation of paying an extra cost of $-g_1(x)$ for extra production of one unit at location x and an extra cost of $g_2(y)$ for creating one unit of demand at y. The extra amounts of mass (commodity/demand) are unknowns of the problem. Our approach to this problem is by taking the limit as $p \to \infty$ to a double obstacle problem (with obstacles g_1, g_2) for the p-Laplacian. In fact, under a certain natural constraint on the extra costs (that is equivalent to impose that the total optimal cost is bounded) we prove that this limit gives the extra material and extra demand needed for optimality and a Kantorovich potential for the mass transport problem involved. We also show that this problem can be interpreted as an optimal mass transport problem in which one can make the transport directly (paying a cost given by the Euclidean distance) or may hire a courier that cost $g_2(y) - g_1(x)$ to pick up a unit of mass at y and deliver it to x. For this different interpretation we provide examples and a decomposition of the optimal transport plan that shows when we have to use the courier.

This is a joint work with J. Mazon and J. Toledo.