Structure of the solution sets to impulsive differential inclusions

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One of methods of studying a dynamics of differential equations (or inclusions) consists in analysis of the Poincaré-Krasnosel'skii translation operator along trajectories $P_T : E \to 2^E$ having the decomposition $P_T = e_T \circ S$, where $S : E \to 2^X$ is a solution map assigning for the problem

$$\begin{cases} \dot{x} \in F(t,x),\ x(t) \in E, \\ \text{possible constraints or impulses} \\ x(t_0) = x_0 \end{cases}$$

the set of solutions $S(x_0)$ in an appropriate function space X, and $e_T(x) = x(T)$ is the evaluation in time T.

For instance, when we are interested in periodic solutions, we would like to look for fixed points of P_T applying, e.g., a fixed point index or the Lefschetz number, etc. This is possible if $P_T = e_T \circ S$ is suitable for the fixed point theory. Hence, the topological structure of values of S is of great importance.

The case without impulses (even with constraints $x(t) \in K$) is relatively well investigated. We focus our attention on problems where sudden jumps $x(t^+) = I(x(t))$ occur in points (t,x), in an extended phase space, satisfying $g_i(t,x) = 0$. Now solutions appear to be discontinuous and the choice of the function space X is not obvious in general.

The first situation we shall examine is: $g_i(t,x) = t - t_i$ so, where impulses are in fixed moments. We shall obtain an R_δ-structure of the solution set for problems on both compact and noncompact intervals. Some recent results in this direction have been published in [1] for functional evolution inclusions $\dot{x} \in A(t)x + F(t,x_t)$ in Banach spaces.

In the second situation we shall consider $g_i(t,x) = t - \tau_i(x)$ (jumps occur in variable times), and an important task is to construct a suitable space X containing all solutions and having a complete normed topology. A proposal of the Banach space X as well as some R_δ-structure results obtained in [2] will be presented.

Some open questions and further research directions will also be given.

References

[1] *G. Gabor, A. Grudzka*: Structure of the solution set to impulsive functional differential inclusions on the half-line. Nonlinear Differ. Equ. Appl. *19* (2012), 609–627.

[2] *A. Grudzka, S. Ruszkowski*: Structure of the solution sets to state-dependent impulsive differential inclusions. To appear.