

Instability of a reaction-diffusion system with unilateral obstacles

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According to a famous idea of A. M. Turing, a possible cause for morphogenesis might be that a stable equilibrium of a reaction system becomes unstable when the diffusion is taken into account. Unfortunately, this effect can only occur if the diffusion speed of the activator is very slow and additionally the diffusion speed of the inhibitor is very fast relatively to that of the activator. However, if there are additional unilateral sources or sinks (no matter how small), one obtains a (non-linearizable) system for which the equilibrium is subject to bifurcation and instability for rather different diffusion speeds, actually even for arbitrarily fast diffusion speed of the activator (even if the inhibitor diffuses relatively slowly). Some corresponding results are presented which are based on joint work with J. Eisner, I.-S. Kim, and M. Kučera, respectively. The proofs of all these results make essential use of degree theory.

References

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