Modelling permanent effects of a temporary stimulus (PETS) in predator-prey and SIR systems

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Modification of behaviour in response to changes in the environment or ambient conditions, based on memory, is typical of the human and, possibly, many animal species. One obvious example of such adaptivity is, for instance, switching to a safer behaviour when in danger, from either a predator or an infectious disease. In human society such switching to safe behaviour is particularly apparent during epidemics. Mathematically, such changes of behaviour in response to changes in the ambient conditions can be described by models involving switching. In most cases, this switching disregards the history and, therefore, memory. Memory can be incorporated into a mathematical model using a phenomenon known as hysteresis, and our goal is to demonstrate why and how hysteresis can arise in context of epidemiology and ecology. We illustrate this idea using a simple SIR (Susceptible, Infectious, Recovered) compartmental model that is applicable in epidemiology, where a Preisach operator is introduced from the first principles [1]. Namely, the population is composed of individuals, which choose a mode of behaviour independently from each other based on the current conditions as well as the individual memory. By treating the total population as an ensemble of such individuals we arrive at Preisach nonlinearity as an adiabatic limit of collective memory. Next, we consider a predator-prey system, where the prey can hide in a refuge in reaction to danger [2], and the reaction is hysteretic. Since the range of responses of the biological species is much more limited than that of humans, we demonstrate which behavioural mechanisms can lead to such behaviour. We derive similar Lotka-Volterra-type equations with Preisach operator. We perform numerical simulations and analysis of steady states of the derived systems to demonstrate how initial states of the memory (Preisach operator) determine the stability of the equilibria. We find several neutrally stable branches of equilibria of the systems (multi-stability), and, moreover, we demonstrate numerically how local dynamics near the branch of equilibria that are always unstable without memory effects can lead to its stabilisation [3].

References

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