Memory in switching two-phenotype populations

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Bacteria that can switch their phenotype in response to variations of the ambient conditions have been the subject of multiple experimental and theoretical studies. In particular, each phenotype can be favored by a certain state of the environment. It has been demonstrated experimentally that the switching response of bacteria can depend on the history of variations of the environmental state. This observation suggested some (simple) type of memory and led to the question whether memory can enhance fitness of the population. In this paper, we examine a simple linear differential model with two phenotypes of bacteria whose switching response to stochastic variations of the environmental input is defined by two threshold parameters, and whose fitness is measured by the average growth rate of the total population. It is shown that the maximal fitness is achieved when the thresholds for switching from one phenotype to the other and backwards are different. We interpret this result as an indication of memory in the optimal switching strategy. Following the standard approach of the theory of systems with hysteresis, we examine the type of memory present by testing the system with a slowly varying input. Due to the difference of the switching thresholds, in this adiabatic limit, the system responds as a non-ideal relay, which is a standard model of binary (bistable) memory. We then consider a partial differential competition model with populations modeled by non-ideal relays with different switching thresholds. Pattern formation in the presence of diffusion is proved.