An optimal shape design approach towards distortion compensation

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By definition, distortion means undesired alterations in workpiece size and shape, which may happen as a side effect at some stage in the manufacturing chain. Assuming that no rate effects occur during the heat treatment, one can tackle this problem mathematically in a hybrid approach. In the first step the optimal microstructure for distortion compensation is computed solving a shape design problem subject to a stationary mechanical equilibrium problem. In the second step an optimal cooling strategy is computed to realize this microstructure. Here, we focus on the first stage and present a novel approach to compute an optimal microstructure or better phase mixture to compensate for distortion.

We assume that the workpiece domain $D \subset \mathbf{R}^3$ consists of a microstructure with two phases in the domains $\overline{\Omega} \subset D$ and $D \setminus \Omega$, separated by a sharp interface Γ. For instance one might think of these two phases as having been created from one parent phase during a heat treatment. To distinguish between the subdomains we introduce the characteristic function $\chi = \chi_\Omega$ of the set Ω, which is equal to 1 for $x \in \Omega$ and 0 otherwise.

As the phases have different densities, we may expect internal stresses along the interface Γ even if no external forces are applied, leading to a deformation of the outer shape. Our aim is utilize this effect by changing the fractions of the two phases or better by changing the interface Γ to achieve a desired outer shape. To this end, we choose a generic cost functional

$$J(u, \chi) = \int_{\Sigma_1} L(u) \mathrm{d}s = \int_{\Sigma_1} |u - u^d|^2 \mathrm{d}s + \alpha \mathcal{P}_D(\Omega)$$

with $\Sigma_1 \subset \Sigma_N$ and a perimeter penalization.

We investigate the resulting optimal shape design problem and derive necessary optimality conditions. For the numerical computation of optimal subdomains we employ a phase field relaxation to the problem. This means, we replace the perimeter term in the cost functional by a Ginzburg-Landau term, i.e.,

$$\int_D \Big(\frac{\gamma\delta}{2} |\nabla \varphi|^2 + \frac{\gamma}{\delta} \psi(\varphi) \Big) \mathrm{d}x,$$

with double-well potential $\psi(\varphi) = c_1(1 - \varphi^2)^2$, $c_1 > 0$. We use an L^2 gradient flow dynamics for φ with an artificial time variable t. The resulting system consists of a parabolic equation for the phase field variable φ coupled to two elliptic equations for the state and its adjoint. We discuss well-posedness of the system and conclude with some numerical simulations.

References

[1] *M. Hintermüller, D. Hömberg, K. Sturm*: Shape Optimization for a Sharp Interface Model of Distortion Compensation. WIAS Preprint No. 1792 (2013).