Shape optimization of the ground state for two phase conductors

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We consider the problem of the optimal distribution of two conducting materials with given volume inside a fixed domain, in order to minimize the first eigenvalue (the ground state) of a Dirichlet operator. It is known, when the domain is a ball, that the solution is radial, and it was conjectured that the optimal distribution of the materials consists in putting the material with the highest conductivity in a ball around the center. We show that this conjecture is not true in general. For this, we consider the particular case where the two conductivities are close to each other (low contrast regime) and we perform an asymptotic expansion with respect to the difference of conductivities. The main result is to show, using asymptotic expansions with respect to ε and to small geometric perturbations of the optimal shape, that the global minimum of the eigenvalue in low contrast regime (for ε small enough) is either a centered ball or the union of a centered ball and of a centered ring touching the boundary, depending on the prescribed volume ratio between the two materials. We also present results for general geometries of the domain. This is a joint work with Carlos Conca and Rajesh Mahadevan.

References

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