Hybrid level set phase field method in shape optimization

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The paper deals with the shape and/or topology optimization of systems governed by the elliptic variational inequalities. Among others, contact phenomenon between the surfaces of the elastic or rigid bodies is described by these inequalities. The structural optimization for a contact problem consists in finding such topology of the domain occupied by the body and/or the shape of its boundary that the normal contact stress along the boundary of the body is minimized. The volume of the body is bounded.

In structural optimization the level set method [1] is employed in numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure, i.e., the position of the boundary of the body is described as an isocountour of a scalar function of a higher dimensionality.

The paper deals with the analysis and numerical solution of the topology optimization of system governed by the variational inequalities using the combined level set and phase field [3], [4] rather than standard level set approach. In the paper two-phase topology optimization problem is formulated in terms of the level set method. Next this problem is regularized using Cahn-Hilliard energy term rather than the perimeter term. The proposed regularization for topology optimization of contact problems is motivated in [2]. Derivatives formulae of the cost functional with respect to the level set function are calculated. Modified reaction-diffusion equation updating the level set function is derived. Necessary optimality condition for this optimization problem is formulated. The finite element method is used to solve the state and adjoint systems. Numerical examples are provided and discussed.

References

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