Degenerate parabolic problems with discontinuous flux

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The talk is devoted to the problems that can be written under the form

$$u_t + (f(x,u) - \lambda(x,u)\pi(x,u)_x)_x = 0$$

where f, λ and π are functions that are continuous in u and piecewise constant in x. The diffusion terms may degenerate, namely $\lambda(\cdot, u)$ is positive and $u \mapsto \pi(x,u)$ is a non-strictly increasing function. Such models appear, for instance, in the context of sedimentation problems (see [3] and references therein).

In the setting of fully degenerate ($\pi \equiv$ const) hyperbolic conservation laws and of non-degenerate ($\pi(x,\cdot)$ strictly increasing) parabolic problems, numerous results were obtained in the recent years (see in particular [2] and references therein). Only few results ([4]) are known in the degenerate hyperbolic-parabolic case we are interested in.

Our goal is to extend the theory of purely hyperbolic conservation law with space-discontinuous flux (see [1]) to the present setting. This includes investigation of different notions of solution to the problem, of its well-posedness and its numerical approximation.

References

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[4] *K. H. Karlsen, N. H. Risebro, J. D. Towers*: L^1 stability for entropy solutions of nonlinear degenerate parabolic convection-diffusion equations with discontinuous coefficients. Skr., K. Nor. Vidensk. Selsk. *2003* (2003), pp. 49.