Multi-dimensional scalar conservation laws with fluxes discontinuous in the unknown and the spatial variable

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We are interested in a scalar conservation law in an arbitrary dimension d with a discontinuous flux F

$$(1)\qquad \begin{aligned} u_t + \operatorname{div} F(x,u) &= 0 \quad \text{in } (0,\infty)\times\mathbb{R}^d,\\ u(0,\cdot) &= u_0 \quad \text{in } \mathbb{R}^d. \end{aligned}$$

The flux is supposed to be a discontinuous function in the spatial variable x and in an unknown function u. Under some additional hypothesis on the structure of possible discontinuities, we formulate an appropriate notion of entropy solution and establish its existence and uniqueness. The structure of the flux function corresponds to the one proposed by [4] for the case of fluxes discontinuous in x. The framework for proving the existence and uniqueness of entropy weak solutions is provided by the studies on entropy measure-valued solutions, cf. [3] and may be viewed as a corollary of the uniqueness theorem for entropy measure-valued solutions. Moreover, the techniques using comparison principle will be presented.

There are numerous applications of considerations on discontinuous flux such as sedimentation process, two phase flow in porous media, modeling of traffic flow and others. The studies are also motivated by an implicit constitutive theory. The understanding of scalar hyperbolic conservation laws with a discontinuous (or multi-valued) flux represent a good starting point for a progress in the mathematical theory for evolutionary problems of elasticity with implicit or discontinuous relations between the Cauchy stress and the deformation gradient.

The talk is based on joint works with M. Bulíček, P. Gwiazda and J. Málek, [1], [2] and also on the work in progress with P. Wittbold and A. Zimmermann.

References

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