Geometric properties of Kahan's method

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There has been a fair amount of work in the last two decades characterizing one step integration-methods for ODEs having certain geometric properties when applied to general Hamiltonian systems: symplectic integrators; energy/integral-preserving integrators; conjugate-to-symplectic integrators. A different point of view arises if we restrict the class of problems from the case of general Hamiltonian functions to the case of polynomial Hamiltonian functions and polynomial Hamiltonian vector fields. Then, it becomes often easier to preserve these geometric properties. A remarkable case is the one of the Kahan's method. We show that Kahan's discretization of quadratic vector fields is equivalent to a Runge-Kutta method. When the vector field is Hamiltonian on either a symplectic vector space or a Poisson vector space with constant Poisson structure, the map determined by this discretization has a conserved modied Hamiltonian and an invariant measure, a combination previously unknown amongst Runge-Kutta methods applied to nonlinear vector fields. This produces large classes of integrable rational mappings in two and three dimensions, explaining some of the integrable cases that were previously known.

This is a joint work with Elena Celledoni, Robert McLachlan and Reinout Quispel.

References

[1] *E. Celledoni, R. I. McLachlan, B. Owren, G. R. W. Quispel*: Geometric properties of Kahan's method. J. Phys. A. *46* (2013), 12 pp.