Degenerating Cahn-Hilliard systems coupled with mechanical effects and complete damage processes

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Complete damage in elastic solids appears when the material loses all its integrity due to high exposure. In the case of alloys, the situation is quite involved since spinodal decomposition and coarsening also occur at sufficiently low temperatures which may lead locally to high stress peaks. Experimental observations on solder alloys reveal void and crack growth especially at phase boundaries. In this talk, we investigate analytically a degenerating PDE system with a time-dependent domain for phase separation and complete damage processes under time-varying Dirichlet boundary conditions. The evolution of the system is described by a degenerating parabolic differential equation of fourth order for the concentration, a doubly nonlinear differential inclusion for the damage process and a degenerating quasi-static balance equation for the displacement field. All these equations are strongly nonlinearly coupled. Because of the doubly degenerating character and the doubly nonlinear differential inclusion, we are confronted with introducing a suitable notion of weak solutions. Based on the work [1], we choose a notion of weak solutions which consists of weak formulations of the diffusion equation and the momentum balance, a variational inequality for the damage evolution and an energy inequality. For the introduced degenerating system, we prove existence of weak solutions in an SBV-framework.

This is a joint work with Christiane Kraus.

References

[1] *C. Heinemann, C. Kraus*: Existence results for diffuse interface models describing phase separation and damage. Eur. J. Appl. Math. *24* (2013), 179–211.