Change of energy caused by crack propagation in 3-dimensional anisotropic solids

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This contribution presents ideas, how crack propagation in three-dimensional solids composed of anisotropic materials can be predicted using the Griffith energy principle. Since the work of Irwin the change of potential energy $\Delta\mathbf{U}$ caused by a straight elongation of a crack in an isotropic two-dimensional homogeneous structure can be expressed in quadratic terms of the stress intensities at the crack tip. This result was generalized in the last decades to anisotropic and also inhomogeneous materials using methods of asymptotic analysis by many authors [1]. With the energy release rate at hand, quasi-static scenarios of crack propagation can be simulated for plane problems [3].

While crack propagation for plane scenarios is widely discussed in the literature [1], this is much more complicated in three dimensions [2]. Mathematical models for crack prediction are based on the asymptotic behavior of the displacements at the crack front. If the crack front is a smooth curve completely contained inside a solid, displacements are of well-known square-root type also in three dimensions. We show, how the asymptotic structure can be calculated, now depending on the geometry of the crack surface. Using methods from asymptotic analysis, we generalize the results from [2] and derive a representation of the change of potential energy caused by a small elongation of a three-dimensional smooth crack surface with arbitrary curvature and torsion:

$$\Delta\mathbf{U} = -\frac{1}{2}t\left(\int_\Gamma h(s)\left(\sum_{i,j=1}^3 K_{i,1}(s)M_{i,j}(\vartheta(s))K_{j,1}(s)\right)\mathrm{d}s\right) + \dots.$$

Here, $K_i(s)$ are the stress intensity factors at arc length s on the crack front Γ and $M_{i,j}$ are so-called local characteristics, depending on the material properties and the geometry of the elongated crack. The quantity $th(s)$ is the length of the crack elongation at the crack front at arc length s to direction $\vartheta(s)$. The number t can be interpreted as a time-like parameter which is always small.

References

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