Uniqueness of solutions of fully implicit nonlinear difference schemes

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When difference schemes are used to show the existence of weak solutions of time-dependent problems, it is convenient to use linearized schemes whenever there is no need to treat nonlinear terms in their fully implicit form. For example, Shinbrot [2, p. 159] used the linearized difference scheme

$$v^k - v^{k-1} + hv^{k-1} \cdot \nabla v^k - h\nu\Delta v^k + h\nabla\pi^k = 0, \quad \nabla \cdot v^k = 0, \quad k = 1, 2, \dots$$

to show the existence of weak solutions of the Navier-Stokes equations

$$\partial_t u + u \cdot \nabla u - \nu\Delta u + \nabla p = 0, \quad \nabla \cdot u = 0.$$

However, if convergence rates are of interest, nonlinear difference schemes can be appropriate as well, for example the following version of the Crank-Nicolson scheme in Heywood/Rannacher [1, p. 366]

$$v^k - v^{k-1} + h\frac{1}{4}(v^{k-1} + v^k) \cdot \nabla(v^{k-1} + v^k) - h\nu\Delta\frac{1}{2}(v^{k-1} + v^k) + h\nabla\pi^k = hf^k, \quad \nabla \cdot v^k = 0.$$

In this talk we discuss the uniqueness of solutions of a prototypical fully implicit nonlinear difference scheme under smallness assumptions on the previous approximation v^{k-1} and the step size h.

References

[1] *J. G. Heywood, R. Rannacher*: Finite-element approximation of the nonstationary Navier-Stokes problem. IV: Error analysis for second-order time discretization. SIAM J. Numer. Anal. *27* (1990), 353–384.

[2] *M. Shinbrot*: Lectures on Fluid Mechanics. Gordon and Breach Science Publishers, New York, 1973.