Existence of global strong solution for Korteweg system with large infinite energy initial data

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This work is devoted to the study of the initial boundary value problem for a general isothermal model of capillary fluids derived by J. E Dunn and J. Serrin (1985), which can be used as a phase transition model. We will prove the existence of local and global (under a condition of smallness on the initial data) strong solutions with discontinuous initial density when $\ln \rho_0$ belongs in the Besov space $B^{\frac{N}{2}}_{2,\infty}(\mathbb{R}^N)$. Our result relies on the fact that the density can be written as the sum of the solution ρ_L associated to linear system and a remainder density $\bar{\rho}$ which is more regular than ρ_L by taking into account the regularizing effects induced on the bilinear convection term. The main difficulty concerns the proof of new estimate of maximum principle type for the linear system associated to the Korteweg system, the proof is based on a characterization of the Besov space in terms of the semi group associated to this linear system. Let also point out that we prove the existence of global strong solution with a smallness hypothesis which is subcritical in terms of the scaling of the equations, it allows us to exhibit a family of large energy initial data for the scaling of the equations providing global strong solution. In particular for the first time up our knowledge we show the existence of global strong solution for some large energy initial data when $N = 2$.

We finish this paper by introducing the notion of quasi-solutions for the Korteweg's system (a tool which has been developed in the framework of the compressible Navier-Stokes equations by the author) which enables us to improve the previous result and to obtain the existence of global strong solution with large initial velocity in $B^{\frac{N}{2}-1}_{2,\infty}$. As a corollary, we get global existence (and uniqueness) for highly compressibleKorteweg system when $N \geq 2$. It means that for any large initial data (under an irrotational condition on the initial velocity) we have the existence of global strong solution provided that the pressure is sufficiently highly compressible.