

A nonuniform dichotomies with different growth rates

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For nonautonomous linear differential equations $v' = A(t)v$ in a Banach space, we consider general exponential dichotomies that extend the notion of (uniform) exponential dichotomy in various ways. Namely, the new notion allows: stable and unstable behavior with respect to growth rates $e^{c\rho(t)}$ for an arbitrary function $\rho(t)$; nonuniform exponential behavior, causing that any stability or conditional stability may be nonuniform; and different growth rates in the uniform and nonuniform parts of the dichotomy. Our objective is fourfold:

- (1) to show that there is a large class of linear differential equations admitting this general exponential behavior;
- (2) to provide conditions for the existence of general dichotomies in terms of appropriate Lyapunov exponents;
- (3) to establish the robustness of the exponential behavior, that is, its persistence under sufficiently small linear perturbations;
- (4) to establish the relationship between such dichotomies and Lyapunov functions.