## Long-term analysis of numerical and analytical oscillations

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Two completely different topics will be adressed in this talk:

- the numerical solution of Hamiltonian systems with linear multistep methods over long times,
- adiabatic invariants in highly oscillatory Hamiltonian differential equations.

In both situations, highly oscillatory solutions are present and their influence to the long-time dynamics of solutions is of interest to us. Whereas in the second situation oscillatory solutions arise from the special form of the differential equation, they are due to the multistep character and the presence of parasitic solution terms in the first situation.

We show that certain symmetric multistep methods, when applied to second order Hamiltonian systems, behave very similar to symplectic one-step methods (excellent long-time energy-preservation, linear error growth for nearly integrable systems). On the other hand, for multiscale systems where harmonic oscillators with several high frequencies are coupled to a slow system, near-preservation of the oscillatory energy over long times is shown without any non-resonance condition.

For the proof of these results the technique of modulated Fourier expansions is used. The surprising fact is that the same ideas that permit to prove the near-preservation of the oscillatory energy in multiscale Hamiltonian systems, can also be applied to get insight into the long-time behavior of symmetric linear multistep methods.

The presented results have been obtained in collaboration with Christian Lubich, David Cohen, Ludwig Gauckler, and Paola Console.

*References*

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