

A quantitative theory in stochastic homogenization

Felix Otto

Max Planck Institute for Mathematics in the Sciences, Germany

otto@mis.mpg.de

In many applications, one has to solve an elliptic equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in a situation where the coefficients are characterized in statistical terms: Their statistics are assumed to be translation invariant and to decorrelate over large distances. As is known by qualitative theory, the solution operator behaves – on large scales – like the solution operator of an elliptic problem with *homogeneous, deterministic* coefficients!

We are interested in several quantitative aspects: How close is the actual solution to the homogenized one—we give an optimal answer, and point out the connections with elliptic regularity theory (input from Nash’s theory, a new outlook on De Giorgi’s theory).

We are also interested in the quantitative ergodicity properties for the process usually called “the environment as seen from the random walker”. We give an optimal estimate that relies on a link with (the Spectral Gap for) another stochastic process on the coefficient fields, namely heat-bath Glauber dynamics. This connection between statistical mechanics and stochastic homogenization has previously been used in opposite direction (i.e. with qualitative stochastic homogenization as an input).

Theory provides a formula for the homogenized coefficients, based on the construction of a “corrector”, which defines harmonic coordinates. This formula has to be approximated in practise, leading to a random and a systematic error. If time permits, we point out optimal estimates of both.

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