From Newtonian system of particles to heat equation

Laure Saint-Raymond
Université Pierre et Marie Curie & Ecole Normale Supérieure, France
Laure.Saint-Raymond@ens.fr

We provide a rigorous derivation of the heat equation as the hydrodynamic limit of systems of hard spheres as the number of particles N goes to infinity and their diameter ε simultaneously goes to 0, in the fast relaxation limit $N\varepsilon^{d-1} \to \infty$ (with the suitable scaling of the observation time and length).

As suggested by Hilbert in his sixth problem, we use the linear Boltzmann equation as an intermediate level of description for one tagged particle in the gas close to global equilibrium.

Our proof relies on the fundamental ideas of Lanford. The main novelty here is the detailed study of the branching process, leading to some explicit estimate on pathological collision trees.

References

[1] *T. Bodineau, I. Gallagher, L. Saint-Raymond*: The linear Boltzmann equation as the low density limit of a system of hard-spheres: Long time convergence and diffusive limits, submitted 2013.

[2] *I. Gallagher, L. Saint-Raymond, B. Texier*: From Newton to Boltzmann: hard spheres and short-range potentials. To appear in Zürich Advanced Lectures in Mathematics 2013.