

# Hierarchical construction of solutions in critical regularity spaces

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We construct uniformly bounded solutions of the equations  $\operatorname{div}(U) = f$  and  $\operatorname{curl}(U) = f$ , for general  $f$ 's in the critical regularity spaces  $L^d(\mathbb{R}^d)$  and, respectively,  $L^3(\mathbb{R}^3)$ . The study of these equations was motivated by recent results of Bourgain & Brezis. The equations are linear but construction of their solutions is not. These constructions are special cases of a rather general framework for solving linear equations in critical regularity spaces. The solutions are realized in terms of nonlinear hierarchical representations,  $U = \sum_j u_j$ , which we introduced earlier in the context of image processing. The  $u_j$ 's are constructed *recursively* as proper minimizers, yielding a multi-scale decomposition of the solutions  $U$ .