

Existence of oscillatory solutions of nonlinear singular ODE

Martin Rohleder

Faculty of Science, Palacký University, Olomouc, Czech Republic

mathin@seznam.cz

We investigate the singular initial problem

$$(p(t)u'(t))' + q(t)f(u(t)) = 0, \quad u(0) = u_0, \quad u'(0) = 0$$

on the half-line $[0, \infty)$. Here $u_0 \in [L_0, L]$, where L_0 , 0 and L are zeros of f , which is locally Lipschitz continuous on \mathbb{R} . Function p is continuous on $[0, \infty)$, has a positive continuous derivative on $(0, \infty)$ and $p(0) = 0$. Function q is continuous on $[0, \infty)$ and positive on $(0, \infty)$. For specific values u_0 we prove the existence and uniqueness of damped solutions of this problem. With additional conditions for f , p and q it is shown that the problem has for each specified u_0 a unique oscillatory solution with decreasing amplitudes.

References

- [1] *M. Cecchi, M. Marini, G. Villari*: On some classes of continuable solutions of a nonlinear differential equation. *J. Differ. Equations* 118 (1995), 403–419.
- [2] *I. Rachůnková, J. Tomeček*: Bubble-type solutions of nonlinear singular problems. *Math. Comput. Modelling* 51 (2010), 658–669.
- [3] *I. Rachůnková, L. Rachůnek, J. Tomeček*: Existence of oscillatory solutions of singular nonlinear differential equations. *Abstr. Appl. Anal.* 2011 (2011), 20 p. Article ID 408525.
- [4] *I. Rachůnková, J. Tomeček, J. Stryja*: Oscillatory solutions of singular equations arising in hydrodynamics. *Adv. Difference Equ.* 2010 (2010), 13 p. Article ID 872160.
- [5] *I. Rachůnková, J. Tomeček*: Strictly increasing solutions of a nonlinear singular differential equation arising in hydrodynamics. *Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods* 72 (2010), 2114–2118.
- [6] *M. Rohleder*: On the existence of oscillatory solutions of the second order nonlinear ODE. *Acta Univ. Palacki. Olomouc, Fac. Rer. Nat., Mathematica* 51 (2012), 107–127.